the system of equations (0.1) and (0.2) can be written in the form

$$\Theta^{1.19}/n^{0.94}+h^2=1,\ \frac{d\varepsilon}{d\xi}=\frac{\sqrt{2\pi}}{n}\xi\left(\frac{dh}{d\xi}-\frac{h}{n}\frac{dn}{d\xi}\right),$$

$$\frac{dh}{d\xi}=-\frac{1}{2\sqrt{2\pi}}\frac{\Theta^{0.86}\varepsilon}{n^{0.86}},\quad q=-\frac{16}{3}\frac{\Theta^{5.14}}{n^{0.86}}\frac{d\Theta}{d\xi}, \qquad (3.1)$$

$$\frac{dq}{d\xi}=\frac{\varepsilon^2}{4\pi}\frac{\Theta^{0.86}}{n^{0.86}}+0.59\xi\left(3\frac{\Theta^{0.19}}{n^{0.06}}\frac{d\Theta}{d\xi}-\frac{\Theta^{1.19}}{n^{1.06}}\frac{dn}{d\xi}\right),\quad \frac{dx}{d\xi}=\frac{1}{n}.$$

From (3.1) and the boundary conditions $h(0) = 1$, $\varepsilon(0) = \text{const}$, $q(0) = \text{const}$, we have the following expansions for $\Theta(\xi)$ and $n(\xi)$:

$$\Theta(\xi)|_{\xi\to0}\sim\xi^{0.23},\ n(\xi)|_{\xi\to0}\sim\xi^{0.51}.$$

The solution of the system (3.1) with the boundary condition $n(\infty) = \infty$ is shown in Fig. 6. The characteristic times corresponding to the start and end (t_i and t_f) of the transition from the regime of Fig. 5 to that of Fig. 6 can be estimated by equating the electric field of Fig. 5 to the fields $E(\infty)$ and $E(0)$ of Fig. 6:

$$t_i=0.0012\ \mu\text{sec}/H_0^{1.12}(\text{MG}),\ t_f=0.0015\ \mu\text{sec}/H_0^{1.12}(\text{MG}).$$

LITERATURE CITED

1. Yu. D. Bakulin and S. P. Kurdyumov, "Some self-modeling problems on the penetration of a magnetic field in a conducting medium" [in Russian], Preprint IPM Akad. Nauk SSSR, No. 61 (1973).
2. S. I. Braginskii, "Transport phenomena in plasmas," in: Problems in Plasma Theory [in Russian], Vol. 1, Atomizdat, Moscow (1963).
3. E. M. Lifshits and L. P. Pitaevskii, Physical Kinetics [in Russian], Nauka, Moscow (1979).
4. V. I. Kogan, "On the effect of radiation by impurities on the energy balance of a plasma pinch," Dokl. Akad. Nauk SSSR, 128, No. 4 (1959).
5. D. E. Post, R. V. Jensen, et al., "Steady-state radiative cooling rates for low-density, high-temperature plasmas," Atomic Data and Nuclear Data Tables, 20, No. 5 (1977).

LOSS OF EQUILIBRIUM AND THE QUASISTATIONARY STATE IN AN EXPANDING RECOMBINING PLASMA

G. M. Zhinzhikov, G. A. Luk'yanov, and N. O. Pavlova

UDC 533.9

Many problems of modern gasdynamics and technical physics are concerned with thermodynamic nonequilibrium states of a medium and conditions with thermodynamic nonequilibrium states of a medium and conditions for obtaining the nonequilibrium state. A problem of this type, whose importance comes from its application to creation of effective plasma lasers [1], is the relaxation of a low-temperature plasma during an adiabatic expansion. The conditions for the loss of equilibrium are well-known for some typical situations in a plasma [2, 3]. As a rule these are situations when the cause of the loss of equilibrium is the steady-state effect of perturbing factors on the parameters of the problem. In the present paper we consider the loss of equilibrium in a nonsteady plasma. Criteria are obtained for the loss of ionization equilibrium, the equilibrium distribution of levels, and thermal equilibrium for the expansion of a plasma which is initially in equilibrium. We also study the closely related conditions for a quasistationary occupation of the excited states in a recombining plasma.

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 15-21, May-June, 1985. Original article submitted March 21, 1984.

0021-8944/85/2603-0313$09.50

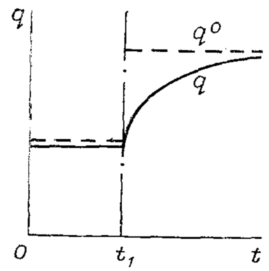

Fig. 1

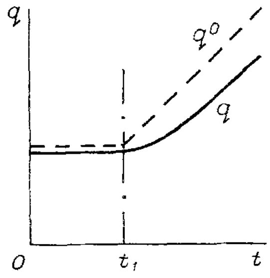

Fig. 2

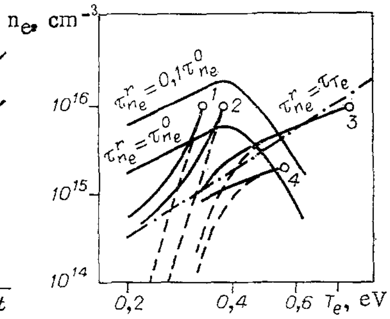

fig. 3

It is known that a parameter q is in equilibrium during a process over a certain time interval τ_0 if at any time in this interval the following condition is satisfied (see [4])

$$\delta/q^0 \ll 1, \tag{1}$$

where $\delta = |q^\circ - q|$; q° and q are the equilibrium (or partial equilibrium) and actual values of the parameter.

If the parameter q satisfies a relaxation equation of the form

$$dq/dt = (|q^0 - q|)/\tau_q, \tag{2}$$

where τ_q is the relaxation time of the parameter, then for an instantaneous change in q° at instant $t = t_1$ by the amount Δq° (as in an idealized shock wave, for example, see Fig. 1) the solution of (2) for τ_q = const and initial equilibrium has the form

$$\delta = \Delta q^0 \exp\left[-(t - t_1)/\tau_q\right]. \tag{3}$$

It then follows that if $\Delta q^\circ/q^\circ$ is not small (not much less than unity) equilibrium is lost in a time of the order of the relaxation time τ_q.

It is known also that equilibrium can be lost for a continuous change in the parameter q°. A typical case ($\delta = 0$ for $t < t_1$, dq°/dt = const for $t \geqslant t_1$) is shown in Fig. 2 (see [5]). The solution of (2) in this case is

$$\delta = \tau_q \frac{dq^0}{dt}\left[1 - \exp\left(-\frac{t - t_1}{\tau_q}\right)\right]. \tag{4}$$

It then follows that we will have an equilibrium process if

$$\tau_q/\tau_q^0 \ll 1,$$

where $\tau_q^\circ = q^\circ/|dq^\circ/dt|$. Similar results are obtained in [6], but for a more rigorous formulation of the conditions for equilibrium than in (1). The inequality

$$\tau_q \geqslant \tau_q^0 \tag{5}$$

can be considered the condition for the loss of equilibrium.

<u>Loss of Ionization Equilibrium.</u> In this case (5) becomes

$$\tau_{n_e} \geqslant \tau_{n_e}^0, \tag{6}$$

where $\tau_{n_e}^\circ$ and τ_{n_e} are the characteristic times for change of the ionization equilibrium and actual concentrations of electrons, i.e., the relaxation time of n_e.

The relaxation time of the electron concentration is the recombination time $\tau_{n_e}^r$. The characteristic time for change of the ionization equilibrium concentration of electrons $\tau_{n_e}^\circ$ is given by the Saha equation, which relates the equilibrium value of the electron concentration n_e° to the electron temperature T_e (which changes as the plasma expands) and the concentration of atoms n_a:

$$\tau_{n_e}^0 = \frac{n_e^0}{\left|\frac{dn_e^0}{dt}\right|} = \frac{1}{\left|\frac{1}{2}\left[\frac{1}{n_a}\frac{dn_a}{dt} + \left(\frac{3}{2} + \frac{I}{kT_e}\right)\frac{1}{T_e}\frac{dT_e}{dt}\right]\right|},$$

where I is the ionization potential.

Since at the instant that ionization equilibrium is lost we have (assuming an adiabatic expansion of the plasma) $(dn_e/dt)^r < 0$ and $dn_e^\circ/dt < 0$, Eq. (6) takes the form

$$\frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^r \geqslant \frac{1}{2}\left[\frac{1}{n_a}\frac{dn_a}{dt} + \left(\frac{3}{2} + \frac{I}{kT_e}\right)\frac{1}{T_e}\frac{dT_e}{dt}\right]. \tag{7}$$

In the case of a simple plasma consisting of atoms, electrons, and single-charged ions

$$n_a = (1 - \alpha)n_e/\alpha,$$

where α is the degree of ionization, and

$$\frac{1}{n_a}\frac{dn_a}{dt} = \frac{\alpha}{1-\alpha}\frac{d}{dt}\left(\frac{1-\alpha}{\alpha}\right) + \frac{1}{n_e}\frac{dn_e}{dt}.$$

Because

$$\frac{1}{\alpha}\frac{d\alpha}{dt} = \frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^r, \quad \frac{dn_e}{dt} = \left(\frac{dn_e}{dt}\right)^f + \left(\frac{dn_e}{dt}\right)^r$$

[$(dn_e/dt)^f$ is the rate of change of the concentration of electrons for frozen ionizations and recombinations, and $(dn_e/dt)^r$ is due to recombinations], then

$$\frac{1}{n_a}\frac{dn_a}{dt} = \frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^f - \frac{\alpha}{1-\alpha}\frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^r.$$

If the effective recombination energy is not too large compared to the enthalpy of the plasma, i.e., $(1/n_e)(dn_e/dt)^f$ and $(1/T_e)dT_e/dt$ are quantities of the same order of magnitude, and also $kT_e/I \ll 1$, then condition (7) takes the form

$$\frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^r \geqslant \frac{1}{2}\left[-\frac{\alpha}{1-\alpha}\frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^r + \frac{I}{kT_e}\frac{1}{T_e}\frac{dT_e}{dt}\right]$$

or, to an excellent approximation (for the purposes of qualitative estimates)

$$\tau_{T_e}/\tau^r_{n_e} \leqslant (1-\alpha)\, I/kT_e, \tag{8}$$

where $\tau_{T_e} = T_e/[dT_e/dt]$ is the characteristic cooling time of the electron component, and this is determined by the conditions of the specific problem. For small values of the degree of ionization, when $1 - \alpha \cong 1$, this condition is given in [7]. Condition (8) gives a qualitatively correct explanation of two features: The lower T_e is, the easier it is to violate ionization equilibrium, and conversely, at high T_e, when $\alpha \cong 1$, it is difficult, and sometimes impossible, to violate the equilibrium. For example, in elements with very different ionization potentials for successive ion multiplicities, there exists a temperature interval such that a decrease in T_e inside the interval does not give (from the Saha equation) a significant change in n_e^o. Therefore in this region, an arbitrarily fast cooling of the electrons does not violate the ionization equilibrium. Examples of such elements are the alkaline earth metals. In Fig. 3 we show the numerical results for a lithium plasma expanding inside a wedge-shaped nozzle with an angular opening of 48° and with a critical diameter height of 2 mm for various (equilibrium) initial parameters and with initial values of the degree of ionization α_0 ranging from 0.01 to 0.999 (points 1 through 4). We see that for the cases considered here, the condition $\tau^r_{n_e} = \tau^o_{n_e}$ (unlike the condition $\tau^r_{n_e} = \tau_{T_e}$ which is usually applied [1]) accurately determines the instant of loss of ionization equilibrium where the actual values of the electron parameters in the nozzle change from the equilibrium values (the dashed curves). Above the curve $\tau^r_{n_e} = 0.1\tau^o_{n_e}$ the deviation from equilibrium does not exceed a few percent for the electron densities.

Loss of the Isothermal Condition in the Plasma. When a plasma which is initially in equilibrium expands, a loss of thermal equilibrium is possible due to recombination heating of the electron components and an insufficient rate of thermalization of the plasma components. In a two-temperature, two-component model of the plasma, the energy equations of the electrons and heavy particles can be written in the form

$$\frac{1}{T_e}\frac{dT_e}{dt} = \frac{2}{3}\frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^f + \frac{Q^r}{n_e(3/2)T_e} - \frac{Q^{\Delta T}}{n_e(3/2)T_e}; \tag{9}$$

$$\frac{1}{T}\frac{dT}{dt} = \frac{2}{3}\frac{1}{n}\frac{dn}{dt} + \frac{Q^{\Delta T}}{(n_e/\alpha)(3/2)T}, \tag{10}$$

where n and T are the concentration and temperature of the heavy particles, Q^r and $Q^{\Delta T}$ are the effective energies of recombinations and elastic exchanges between the electrons and heavy particles per unit volume and time, respectively.

From the structure of Eqs. (9) and (10) we see that the deviation from thermal equilibrium occurs in the background of identical temperature changes of the components during the adiabatic expansion $[(1/n_e)(dn_e/dt)^f = (1/n)dn/dt]$; in this case only the relaxation term proportional to the temperature difference ΔT is present in (10). Therefore the loss of thermal equilibrium depends on the rate of temperature relaxation of the heavy component to that of the electrons, which is changing because of recombination heating and cooling due to elastic exchanges. In this approach it is assumed that $\tau_T^o = \tau_{T_e}$ and the condition for the loss of thermal equilibrium follows from (9) and (10) and takes the form $Q^{\Delta T} \leqslant Q^r/(1 + \alpha)$.

The relation $Q^{\Delta T} = Q^r$ is the condition for a quasistationary change in the electron temperature, in the case when it is determined by the balance of energies acquired by the electrons during recombinations and lost by them in elastic exchanges with the heavy particles [8]. It then follows that the electron temperature change can be quasistationary only for a nonisothermal plasma. Since $Q^r \sim I^*/\tau_{n_e}^r$ and $Q^{\Delta T} \sim (3/2)kT_e/\tau^{\Delta T}$ (I^* is the effective recombination energy and $\tau^{\Delta T}$ is the characteristic thermalization time for elastic collisions), the condition for the loss of thermal equilibrium takes the form

$$\tau^{\Delta T}/\tau^r_{n_e} \geqslant (3/2)\,(1+\alpha)\,kT_e/I^*.$$

Loss of the Relative Equilibrium of the Population Densities of Levels. In an expanding plasma with an equilibrium (at the initial instant of time) distribution of excited states, the Boltzmann equilibrium can be lost due to the recombination flux $(dn_e/dt)^r$ and also due to a change in the electron temperature T_e. Geometrically the expansion does not lead to the loss of the equilibrium distribution because identical changes in the population densities of the levels does not change the distribution temperature.

We consider the equation for the change of the population density of the k-th excited state n_k in the framework of a modified diffusion approximation [2]

$$\partial n_k/\partial t = n_{k-1}z_{k-1,k} - n_k(z_{k,k+1} + z_{k,k-1} + A_k) + n_{k+1}(z_{k+1,k} + A_{k+1,k}), \qquad (11)$$

where n_k are the population densities of the levels, the z_k are the effective probabilities for single-quantum collisional transitions (these probabilities take into account exactly (in the framework of the model) the single-quantum transitions and approximately include other transitions), and A_k are the probabilities of radiative transitions.

For a recombining plasma, (11) can be written in the form

$$\partial n_k/\partial t = n_{k-1}z_{k-1,k} - n_k(z_{k,k-1} + A_k) + j_k, \qquad (12)$$

where $j_k = n_{k+1}(z_{k+1,k} + A_{k+1,k}) - n_k z_{k,k+1}$ is the resultant electron flux into level k "from above."

In order that the equilibrium values of the population densities of states k and k − 1 are maintained during the expansion of the plasma (for $A_k/z_{k,k-1} \ll 1$)

$$n_k^0/g_k = (n_{k-1}/g_{k-1})\exp(-\Delta E_{k,k-1}/kT_e) \qquad (13)$$

($\Delta E_{k,k-1}$ is the energy gap between the levels, g_k and g_{k-1} are the statistical weights) the steady-state value of the population density of level k (given by (12) with the left hand side replaced by zero) must be equal to the equilibrium population density of the level and the relaxation time of the population density of level k must be much smaller than the characteristic time of change of the equilibrium value of the population density.

The first condition will be satisfied if $j_k \ll n_k z_{k,k-1}$ [this follows from (12)]. When $j_k \cong -(dn_e/dt)^r$ this inequality takes the form

$$\tau_{n_k}/\tau^r_{n_e} \ll n_k/n_e, \qquad (14)$$

where $\tau^r_{n_e}$ is the characteristic recombination time.

In a decomposing plasma in the steady state, condition (14) gives restrictions on the changes of the plasma parameters such that the relative equilibrium of the levels will be maintained. The second condition means that $\tau_{n_k} \ll \tau^o_{n_k}$.

The rate of change of the population density dn^o_k/dt in equilibrium is obtained as in the derivation of the condition for ionization equilibrium. We differentiate the equilibrium variable (in this case (13)) and it then follows that

$$\frac{1}{n_k}\frac{dn_k^0}{dt} = \frac{1}{n_{k-1}}\frac{dn_{k-1}}{dt} + \frac{\Delta E_{k,k-1}}{kT_e}\frac{1}{T_e}\frac{dT_e}{dt}.$$

If $dn_{k-1}/dt \leqslant 0$ (for $k - 1 \geqslant 2$) which is usually the case (see (14)), then the necessary condition to maintain the relative equilibrium of the population densities is

$$\tau_{n_k}/\tau_{T_e} \ll kT_e/\Delta E_{k,k-1}. \tag{15}$$

Condition for Quasistationary Population Densities in a Recombining Plasma. It is known that for wide variations in the rates of change of the plasma parameters, quasistationary values of the population densities of the excited states which are close to the steady-state values n_k^∞ [given by (11) with the left-hand side replaced by zero] can occur. The quasistationary condition $dq/dt = 0$ is obviously a mathematical idealization of the process in which the relaxation time τ_q of the parameter q is much smaller than the characteristic time of change τ_q^∞ of the steady-state value of the parameter:

$$\tau_q/\tau_q^\infty \ll 1. \tag{16}$$

It follows from (16) that the determination of conditions for the quasistationary state reduces to finding the values of τ_q and τ_q^∞ in specific cases.

The relaxation time τ_q is determined by linearization of an appropriate dynamical equation [2] (in this case equation (11)) and the characteristic time of change of the steady value of the parameter τ_q^∞ is given in terms of the steady value of the parameter q (from the logarithmic derivative of n_k^∞ from (11) with zero-left hand side).

We determine the condition for a quasistationary distribution of excited states in a recombining, cooling plasma in two limiting cases: small recombination flux $j_k \ll n_k^0 z_{k,k-1}$ and large recombination flux $j_k \gg n_k^0 z_{k,k-1}$.

In the first case $n_k^\infty = n_k^0$ and the condition for a steady value of the population density of level k reduces to the condition (15) to maintain relative equilibrium:

$$\tau_{n_k}/\tau_{T_e} \ll kT_e/\Delta E_{k,k-1}, \tag{17}$$

where $\tau_{n_k} = 1/(z_{k,k-1} + z_{k,k+1})$.

As an example, in Fig. 4 we show the region of a hydrogen plasma (the shaded triangle in the lower portion) in which the quasistationary condition for the occupation of the level k = 2 is violated for a characteristic cooling time of the plasma given by $\tau_{T_e} \leqslant 2 \cdot 10^{-8}$ sec. In the upper portion of Fig. 4 we show the region where the steady-state condition is violated from [1]. Because the coronal approximation is valid for the levels k = 1 and 2 of hydrogen for the regions of the parameters shown in Fig. 4, the equilibrium value of the population density of the level k = 2 was determined with the help of this model. Then the condition that the recombination flux be small is obviously $j_k \ll n_k^\infty A_{21}$.

In the second case the steady-state solution for n_k will be $n_k^\infty = j_k/z_{k,k-1}$ and thus the relative rate of change of the steady-state value is given by

$$\frac{1}{n_k^\infty}\frac{dn_k^\infty}{dt} = \frac{1}{j_k}\frac{dj_k}{dt} - \frac{1}{z_{k,k-1}}\frac{dz_{k,k-1}}{dt}. \tag{18}$$

Now $z_{k,k-1} \sim n_e T_e^{-0.5}$ [2] and for $j_k \cong -(dn_e/dt)^r$ the approximation $j_k \sim n_e^3 T_e^{-4.5}$ is correct. Hence (18) takes the form

$$\frac{1}{n_k^\infty}\frac{dn_k^\infty}{dt} = 2\left(\frac{1}{n_e}\frac{dn_e}{dt} - 2\frac{1}{T_e}\frac{dT_e}{dt}\right). \tag{19}$$

For a nonmoving plasma we have $(1/n_e)dn_e/dt = (1/n_e)(dn_e/dt)^r$. The quasistationary condition in this case will depend on the relation between $\tau_{n_e}^r$ and τ_{T_e}. From (16) and (19) when

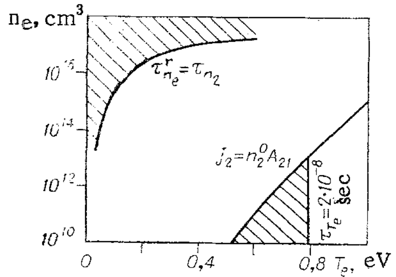

Fig. 4

$\tau^r_{n_e} \ll \tau_{T_e}$ this relation has the form

$$\tau_{n_k}/\tau^r_{n_e} \ll 1,$$

and in the opposite case ($\tau_{T_e} \ll \tau^r_{n_e}$)

$$\tau_{n_k}/\tau_{T_e} \ll 1.$$

We consider how the quasistationary condition changes when the plasma expands.

When $j_k \ll n^0_k z_{k,k-1}$ the quasistationary condition and the condition that equilibrium be maintained are equivalent; therefore, expansion of the plasma does not violate the quasistationarity condition, as noted above.

When $j_k \gg n^0_k z_{k,k-1}$, the situation is different. Expansion of the plasma causes a change in the recombination flux j_k and in the decay probability $z_{k,k-1}$, changing the steady-state value of the population density n^∞_k and therefore directly changing the population densities, which can obviously be considered as changing the relaxation rates of the levels.

For a rapid expansion, when the changes in the temperature and concentration of the electrons are related adiabatically (with the adiabatic index $\gamma = 5/3$), Eq. (19) has the form

$$\frac{1}{n^\infty_k}\frac{dn^\infty_k}{dt} = -\frac{2}{3}\frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^f,$$

and it follows that the steady-state values of the population densities increase for a rapid expansion. Then the quasistationary condition (16) can be written in the form

$$\frac{1}{\tau_{n_k}} + \frac{1}{n_k}\left(\frac{dn_k}{dt}\right)^f \gg -\frac{2}{3}\frac{1}{n_e}\left(\frac{dn_e}{dt}\right)^f$$

or since $(1/n_k)(dn_k/dt)^f = (1/n_e)(dn_e/dt)^f$:

$$\tau_{n_k}/\tau^f_{n_e} \ll 1.$$

For the case when the characteristic times of expansion and recombination are of the same order of magnitude, the quasistationary condition depends on which term on the right-hand side of (19) is dominant.

Therefore, the conditions given here for the existence of a quasistationary population of levels in an expanding plasma lead to a correct estimate of the validity in using a quasistationary model for the population of levels in problems of this type.

The authors thank V. I. Kislov for permission to use the results of his numerical calculations.

LITERATURE CITED

1. L. I. Gudzenko and S. I. Yakovlenko, Plasma Lasers [in Russian], Atomizdat, Moscow (1978).
2. L. M. Biberman, V. S. Vorob'ev, and I. T. Yakubov, Kinetics of Nonequilibrium Low-Temperature Plasmas [in Russian], Nauka, Moscow (1982).
3. A. A. Kudryavtsev and V. N. Skrebov, "On criteria for an equilibrium distribution of atoms over excited states in a low-temperature plasma," Teplofiz. Vys. Temp., 19, No. 6 (1981).
4. Ya. B. Zel'dovich and Yu. I. Raizer, The Physics of Shock Waves and High-Temperature Hydrodynamical Phenomena [in Russian], Nauka, Moscow (1966).
5. V. A. Beserkerskii and E. P. Popov, Theory of Control Systems [in Russian], Nauka, Moscow (1975).
6. O. Yu. Polyanskii, "On the possible types of nonequilibrium flow," ChMMSS, 7, No. 3 (1976).
7. G. A. Luk'yanov, N. O. Pavlova, and G. M. Zhinzhikov, "Nonequilibrium phenomena in rapidly expanding plasma flows," in: Proc. XV International Conference on Phenomena in Ionized Gases: Contributed Papers, Part II, Minsk (1981).
8. G. A. Luk'yanov, "On a recombining plasma-dynamical laser in a freely expanding hydrogen plasma jet," Zh. Tekh. Fiz., 46, No. 4 (1976).